Math 2263 Section 10 Quiz 2

Name: ____________________________

Time limit: 15 minutes

1. (10 points) Reduce the equation

$$4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0$$

to one of the standard forms and **state** which kind of quadric surface it represents.

Solution: Completing squares, we get

$$\begin{aligned} 0 &= 4x^2 + (y^2 - 4y + 4 - 4) + 4(z^2 - 6z + 9 - 9) + 36 \\ &= 4x^2 + (y-2)^2 - 4 + 4(z-3)^2 - 36 + 36\,. \end{aligned}$$

Rewriting,

$$4x^2 + (y-2)^2 + 4(z-3)^2 = 4\,.$$

Finally we divide by 4 to get the standard form

$$x^2 + \frac{(y-2)^2}{4} + (z-3)^2 = 1\,,$$

which is an ellipsoid.

2. (10 points)Show that the limit

$$\lim_{(x,y)\to(0,0)} \frac{y^3 \sin^2 x}{xy^4 + x^5}$$

does not exist. **Explicitly** state along which paths you are evaluating the limit.

Solution: Approaching $(0,0)$ along the x-axis $y = 0$, the numerator vanishes (while the denominator is nonzero), so the limit is zero.
Approaching $(0,0)$ along the line $y = x$, the limit becomes

$$\lim_{x\to 0} \frac{x^3 \sin^2 x}{2x^5} = \lim_{x\to 0} \frac{\sin^2 x}{2x^2} = \frac{1}{2} \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = \frac{1}{2}$$

because $\lim_{x\to 0} \frac{\sin x}{x} = 1$.
Since $\frac{1}{2} \neq 0$, the limit does not exist.

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3. (10 points) Find the domain of the function $G(x, y) = 4 + \sqrt{25 - x^2}$ (in the form $\{(x, y) : \dots\}$) and then sketch the domain in the xy-plane.

Solution: The only restriction is imposed by the square root, so the domain is

$$\{(x, y) : 25 - x^2 \geq 0\} = \{(x, y) : x^2 \leq 25\} = \{(x, y) : -5 \leq x \leq 5\},$$

which is a vertical strip on the xy-plane.

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